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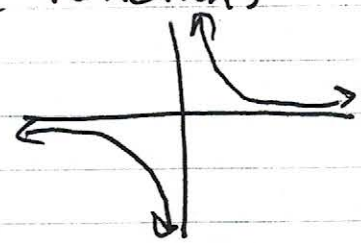
4.2 Graphing Rational Functions

Parent Function

$f(x) = \frac{1}{x}$



$f(x) = \frac{P(x)^m}{Q(x)^n}$

1. Vertical Asymptotes $Q(x) = 0$
2. Horizontal Asymptotes
   a) If $m < n$ then $y = 0$
   b) If $m > n$ then no asymptote
   c) If $m = n$ then use the coefficants

ex) Find the vertical and horizontal asymptote

$f(x) = \frac{2x+1}{3x-4}$

1. Vertical

$3x - 4 = 0$
$+4 \quad +4$
$3x = 4$
$x = 4/3$

2. Horizontal

$\frac{2x+1}{3x-4}$ $m = n$

$y = 2/3$

ex) Find the Vertical and horizontal Asymptote

$f(x) = \frac{4x^2-9}{x^2+2x-15}$

1. Vertical

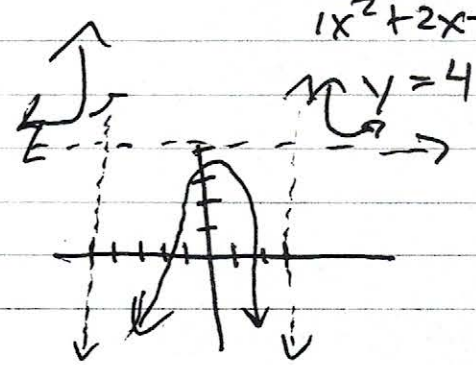
$x^2 + 2x - 15 = 0$ (factor)
$(x+5)(x-3) = 0$
$x + 5 = 0 \quad x - 3 = 0$
$x = -5 \quad x = 3$

2. Horizontal

$\frac{4x^2-9}{1x^2+2x-15}$ $m = n$

$y = 4$

b) Graph



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4.3 Multiply and Divide Rational Functions

Ways to factor

1. GCF
2. Chart (3 Terms)
3. Difference of 2 squares (2 Terms)
4. Cubes (2 Terms)
5. Grouping (4 Terms)

ex) Write an equivalent Expression

$$\frac{x^3-5x^2-24x}{x^3+x^2-72x}$$

1. x^3-5x^2-24x (GCF = x)
   $x(x^2-5x-24)$ (chart)
   $x(x-8)(x+3)$

2. x^3+x^2-72x (GCF = x)
   $x(x^2+x-72)$ (chart)
   $x(x+9)(x-8)$

3. $\frac{\cancel{x}\cancel{(x-8)}(x+3)}{\cancel{x}(x+9)\cancel{(x-8)}} = \frac{x+3}{x+9}$

b) $\frac{x^2+2x+1}{x^3-2x^2-3x}$

1. x^2+2x+1 (chart)
   $(x+1)(x+1)$

2. x^3-2x^2-3x (GCF)
   $x(x^2-2x-3)$ (chart)
   $x(x-3)(x+1)$

3. $\frac{(x+1)\cancel{(x+1)}}{x(x-3)\cancel{(x+1)}} = \frac{x+1}{x(x-3)}$

ex) Find the domain

a) $\frac{x^3+4x^2-x-4}{x^2+3x-4}$

$x^2+3x-4=0$ (chart)
$(x+4)(x-1)=0$
$x+4=0 \quad x-1=0$
$x=-4 \quad x=1$
$\boxed{x \neq -4 \quad x \neq 1}$

b) $\frac{x+2}{x^2-9}$

$x^2-9=0$ (Diff 2 squares)
$(x-3)(x+3)=0$
$x-3=0 \quad x+3=0$
$\cancel{x \neq 3} \quad -3 \; -3 \quad x \neq -3$

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4.3 B Multiply and Divide

ex) $\frac{2xy}{z} \cdot \frac{3x^2}{4yz} = \frac{6x^3y}{4yz^2} = \frac{3x^3}{2z^2}$

b) $\frac{(x-y)^2}{x+y} \cdot \frac{10x+10y}{(x^2-y^2)}$

1. $(x-y)(x-y)$
2. $x+y$
3. $\frac{10(x+y)}{(x-y)(x+y)}$
4. $\frac{(x-y)(x-y)}{(x+y)} \cdot \frac{10\cancel{(x+y)}}{\cancel{(x-y)}\cancel{(x+y)}} = \frac{10(x-y)}{(x+y)}$

c) $\frac{x+2}{x^4-16} \cdot \frac{x^3+4x^2-12x}{1}$

1. $x+2$
2. $(x^4/16)$ diff 2 squares
   $(x^2-4)(x^2+4)$ diff 2 squares
   $(x-2)(x+2)(x^2+4)$
3. x^3+4x^2-12x (GCF)
   $x(x^2+4x-12)$ (chart)
   $x(x+6)(x-2)$
4. $\frac{\cancel{x+2}}{\cancel{(x-2)}\cancel{(x+2)}(x^2+4)} \cdot \frac{x(x+6)\cancel{(x-2)}}{1}$

$\frac{x(x+6)}{x^2+4}$

Divide (Skip, flip, multiply

ex) $\frac{2x^2-12x}{x+5} \div \frac{x-6}{x+5}$

(GCF) $\frac{2x^2-12x}{x+5} \cdot \frac{x+5}{x-6}$

$\frac{2x\cancel{(x-6)}}{\cancel{x+5}} \cdot \frac{\cancel{x+5}}{\cancel{x-6}} = 2x$

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4.4 Add and Subtract Rationals

Find a common denominator

1. Factor denominator
2. Find whats missing
3. multiply Numerator
4. Combine Numerator ✱ Check if + or − ✱
5. Factor Numerator
6. Cancel like Terms

ex) $\frac{x}{x+4} + \frac{5}{x+4} = \frac{x+5}{x+4}$

✱ Denominators the same ✱

b) $\frac{x+3}{x^2-1} + \frac{2}{x^2-3x+2}$

	1. Factor bottom	2. missing	3. multiply Top
$\frac{x+3}{x^2-1}$	$\frac{x+3}{(x-1)(x+1)}$	$\left(\frac{x-2}{x-2}\right)$	$\frac{x^2-2x+3x-6}{(x-1)(x+1)(x-2)}$
$+ \frac{2}{x^2-3x+2}$	$\frac{2}{(x-2)(x-1)}$	$\left(\frac{x+1}{x+1}\right)$	$\frac{2x+2}{(x-2)(x-1)(x+1)}$

4. $\frac{x^2+3x-4}{(x-1)(x+1)(x-2)}$

5. $\frac{(x+4)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)(x-2)} = \frac{x+4}{(x+1)(x-2)}$

Find domain (use Step 4)

$x-1=0$ $\quad$ $x+1=0$ $\quad$ $x-2=0$

$x \neq 1$ $\quad$ $x \neq -1$ $\quad$ $x \neq 2$

over →

Subtraction

ex) $\frac{x+1}{x^2-6x-16}$ 1. $\frac{x+1}{(x-8)(x+2)}$ 2. $\left(\frac{x+4}{x+4}\right)$

$\frac{x+1}{x^2+6x+8}$ $\frac{x+1}{(x+4)(x+2)}$ $\left(\frac{x-8}{x-8}\right)$

3. $\frac{(x+1)(x+4)}{(x-8)(x+2)(x+4)} = \frac{x^2+4x+1x+4}{(x-8)(x+2)(x+4)}$ 4. $\frac{12x+12}{(x-8)(x+2)(x+4}$

$\frac{(x+1)(x-8)}{(x-8)(x+2)(x+4)} = \frac{x^2-8x+1x-8}{(x-8)(x+2)(x+4)}$

5. $\frac{12(x+1)}{(x-8)(x+2)(x+4)}$

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4.5 Solve Rational Equations

ex) $\frac{1}{2}x + \frac{2}{5} = \frac{9}{10}$ LCD: 10

$\frac{1}{2}x(10) + \frac{2}{5}(10) = \frac{9}{10}(10)$

$5x + 4 = 9$

$5x = 5$

$x = 1$

b) $\frac{1}{x-5} + \frac{x}{x-3} = \frac{2}{(x^2-8x+15)}$ (Factor chart)

$\frac{1}{\cancel{(x-5)}}\left(\cancel{(x-5)}(x-3)\right) + \frac{x}{\cancel{x-3}}\left((x-5)\cancel{(x-3)}\right) = \frac{2}{\cancel{(x-5)}\cancel{(x-3)}}\left(\cancel{(x-5)}\cancel{(x-3)}\right)$

LCD: $(x-5)(x-3)$

$x-3 + x(x-5) = 2$

$x-3+x^2-5x = 2$ (set $=0$)

$\quad -2 \qquad\qquad -2$

$x-5+x^2-5x = 0$ (standard form)

$x^2-5x+x-5=0$

$x^2-4x-5=0$ (Factor chart)

$(x-5)(x+1)=0$

$x-5=0 \qquad x+1=0$

$x=5 \qquad x=-1$

* Check extraneous solutions *

over →

$\frac{1}{5-5} = \frac{1}{0}$ since denominator $=0$ then 5 is not a solution

c) $\frac{5x}{x-2} = 7 + \frac{10}{x-2}$ LCD: $x-2$

$\frac{5x}{\cancel{x-2}}(\cancel{x-2}) = 7(x-2) + \frac{10}{\cancel{x-2}}(\cancel{x-2})$

$5x = 7(x-2) + 10$

$5x = 7x - 14 + 10$

$5x = 7x - 4$

$-7x \quad -7x$

$-2x = -4$

$x = 2$ ✱ Check ✱

Since 2 does not work then No solution

d) $\frac{1}{x+4} = \frac{2}{1}$

$2(x+4) = 1$

$2x + 8 = 1$

$-8 \quad -8$

$2x = -7$

$x = -7/2$ ✱ Check ✱